SECTION A

1. A watermelon is 99% water. Alexandra buys a 1kg watermelon. She leaves it out in the sun so that by the time she wants to eat it, it has shrivelled down to only consist of 98% water. What is the total weight, in grams, of the watermelon now?

2. What is the largest number of pieces you can cut a cake into by using only 3 straight cuts with your knife?

   A 4 **B** 5 **C** 6 **D** 7 **E** 8

3. Yun wanted to cook 3 pancakes. Her pan could only fit 2 pancakes at a time, and Yun wanted both sides of each pancake to be cooked. If it takes 1 minute to cook one side of the pancake, what is the shortest amount of time in which she could cook her pancakes?

   A 3min **B** 4min **C** 5min **D** 6min **E** 8min

4. If $1 - 1 = 0$, $2 - 1 = 3$, $3 - 1 = 8$ then what is $5 - 1$?

5. In a park there is twice as many dogs as people, and twice as many people as snakes. The total number of eyes and legs in the park is 510. How many dogs are there?

6. What is the remainder when 11^{2019} is divided by 100?

7. There is a store on Mile End Road that sells one type of chocolate bar. Each bar costs £1, and contains inside it a golden ticket. The store allows you to trade in 3 golden tickets for a new chocolate bar. Lisa, who loves chocolate, heads to the store with £405 in hand. How many chocolate bars can she obtain in total?

8. Turtles A, B and C are having a 100m race. Each turtle moves with a constant speed. Turtle A beats turtle B by 10m, and turtle B beats turtle C by 10m. By how many meters does turtle A beat turtle C?

   A 15 **B** $\frac{50}{\pi}$ **C** $\frac{2100}{121}$ **D** 19 **E** 20

9. A clock is showing the time to be twenty past one. How many degrees are there between the hours hand and the minutes hand.

   A 75˚ **B** 80˚ **C** 85˚ **D** 90˚ **E** 95˚

10. When Adam and Bob stand on an incorrectly calibrated scale it reads the weight 127kg. When Adam stands on it by himself it reads 72kg. How much does Bob weigh?

11. Adam calculates the product $1 \times 2 \times 3 \times \ldots \times 75$. He gets a large number which has a long string of zeroes at the end. How many zeroes does the string have?

A 15 **B** 16 **C** 17 **D** 18 **E** 19

12. From the numbers $\{1, 2, \ldots, 10\}$, Miriam forms the following sequence:

$$8, 5, 4, 9, 1, 7, 6, \ldots$$

What should the next three numbers in her sequence be?

A $10, 3, 2$ **B** $3, 2, 10$ **C** $2, 10, 3$ **D** $10, 2, 3$ **E** $2, 3, 10$

13. A set contains 7 numbers. The average of the 4 smallest numbers is 10 whilst the average of the 4 largest numbers is 30. The average of all the numbers in the set is 20. What is the median of the numbers?

A 15 **B** 18 **C** 20 **D** 25 **E** 27

14. For how many integers n do we have that $4000\left(\frac{2}{5}\right)^n$ is an integer?

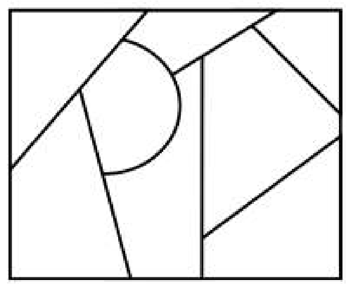
A 3 **B** 4 **C** 6 **D** 8 **E** 9

15. The four sisters Ashley, Billy, Carmen and Dora have distinct heights.
Ashley: "I am not the tallest or shortest."
Billie: "I am not the shortest."
Carmen: "I am the tallest."
Dora: "I am the shortest."

Exactly one of the sisters always lies. Who is the tallest?

SECTION B

1. The map below represents 8 towns. Using 4 colours, in how many ways can you colour the map such that no two towns that share a border have the same colour.



A 192 **B** 384 **C** 576 **D** 768 **E** 1152

2. Alice and Bob asked Clarice which number was her favourite. She revealed to Alice the first digit and to Bob the second. She told both of them that her favourite number was one of the follwing:

$$35, 36, 39, 57, 58, 74, 76, 94, 95, 97$$

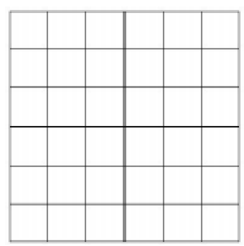
Looking at the list, Alice said "I don't know which number is Clarice's favourite, but I am certain that Bob doesn't either". Then Bob replies "At first I didn't know Clarice's favourite number, but now I know", to which Alice said "Then now I know as well". What is Clarice's favourite number?

3. Rami, Sara and Tom were playing a game. They would place their names into a hat, and starting with Rami, then Sara, then Tom, would take it in turns drawing a name from the hat. If one's own name is drawn, the rule is to put the name back into the hat and draw again until one draws another's name. If the last person remaining draws their own name, the whole game is nullified and they start over. They repeat these rules until they successfully each have a different person's

name in their hand. What is the probability that Rami gets Tom's name after the game is complete?

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** $\frac{2}{3}$ **D** $\frac{3}{4}$ **E** $\frac{3}{7}$

4. How many rectangles are there that contain at least 3 cells, in the following grid:



A 84 **B** 210 **C** 320 **D** 345 **E** 441

5. How many positive integers k less than 2019 are there which satisfy that $k = 2^m + 2^n$ for some integers m and n. For example $24 = 2^4 + 2^3$.

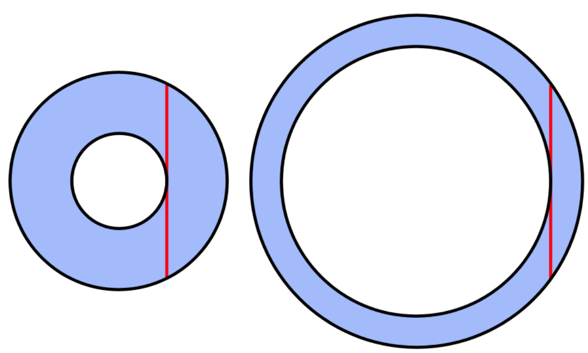
A 11 **B** 55 **C** 66 **D** 202 **E** 1009

6. You own 25 horses, each of which runs at a different constant speed. You want to determine which 3 horses are the fastest in the group. You have a running track with 5 lanes. In each race you can put up to 5 horses in competition. However you don't have a stopwatch so you are only able to calculate the horses' relative ranking but not their speeds. What is the minimum number of races you need to conduct in order to determine the 3 fastest horses?

7. Every Saturday from 5pm to 6pm Stuart and Sam play tennis. At exactly 6pm Stuart's wife picks him up and they drive home. One specific Saturday Stuart arrived for tennis but remembered that Sam had told him he wouldn't be coming that day. So at 5pm Stuart starts walking home in the direction that his wife takes when driving. Some time later he meets his wife along the way. She picks him up and drives home, arriving 20 minutes earlier than usual. For how many minutes was Stuart walking?

   A 30 **B** 40 **C** 45 **D** 50 **E** 55

8. The following picture shows two (flat) rings where the red vertical lines have the same length in both pictures:



   Which of the following is true?

   A The left ring's shaded area is 4 times smaller than the right ring's one

   B The left ring's shaded area is 2 times smaller than the right ring's one

C Both areas are the same

D The left ring's shaded area is 2 times bigger than the right ring's one

E The left ring's shaded area is 4 times bigger than the right ring's one

9. A merchant is travelling to Venice, and wants to take with him his classic balance scale (as shown in the picture below). He needs to be able to weigh every whole number weight from 1kg to 40kg. However, because he has luggage restrictions, he doesn't want to carry with him 40 1kg blocks, but wants to minimize the number of blocks. What is the smallest number of blocks he has to carry, and what should each of their weights be?

10. Aisha, Benjamin, Chloe and Dimitri were out walking at night when they arrived at a treacherous bridge. The bridge could carry at most two people at any time, and to cross it one needs a flashlight. The group only had one flashlight.

    Aisha is the quickest in the group and is able to cross the bridge in 1 minute. Benjamin needs 2 minutes, Chloe 5 minutes and Dimitri 10 minutes. What is the shortest time in which they could all cross the bridge? (Assume that when two people walk together, they walk at the pace of the slower person. The flashlight cannot be thrown across the bridge)!

11. Alan, Barack, and Cee take it in turns choosing numbers between 0 and 1. The first person to land a number that lies in between the previous two numbers wins the game. Alan declares his strategy to be that he will choose, at every step, a random number between 0 and 1. Barack says he will choose at every step a random number between $\frac{1}{2}$ and $\frac{2}{3}$. Given this information, which number should Cee choose to maximize her chances of winning?

    A $\frac{1}{2}$ **B** $\frac{13}{24}$ **C** $\frac{7}{12}$ **D** $\frac{5}{8}$ **E** $\frac{2}{3}$

12. Consider all the possible arrangements of the word BETWEEN. We wish to make a "dictionary" for these words, starting with the word BEEENTW and ending with WTNEEEB. In which position in our dictionary will the word BETWEEN lie?

    A 46 **B** 49 **C** 55 **D** 57 **E** 65

13. A certain positive integer n has the property that the sum of its digits is 1274. Which of the following alternatives could be the sum of the digits of $n + 1$?

    A 1 **B** 3 **C** 942 **D** 1238 **E** 1265

14. Fatima writes down the 79 digit number 123456789101112 . . . 4344 by sticking together all the numbers from 1 to 44. What is the remainder when her number is divided by 45?

    A 0 **B** 1 **C** 4 **D** 9 **E** 44

15. Five pirates gathered to split the 100 gold coins that they had stolen. They went in order of rank, with the captain first proposing how to split the coins. After hearing the proposal, all pirates would vote. If half or more voted for the proposal, it went through, otherwise the captain is killed and the second highest ranking pirate proposes a split, to which again they will all vote, etc.

    If all pirates are perfect logicians, how many gold coins can the captain guarantee to keep?

SECTION C

1. Three high school students, named Alice, Bob, and Charlie were taking their final examinations. All three had to take exactly the same exams. Points for these exams were awarded in a rather strange way. In every exam that you would come first in you were awarded a fixed x number of points. In every exam that you would come second you were awarded a fixed y number of points, and in every exam that you would come third you were awarded a fixed z number of points. Assume $x > y > z > 0$, and all the points are whole numbers. The teachers of the high school didn't believe in ties so in no exam did the students tie.

   At the end of the examinations, and after all the marks were out, Alice had come first over-all with 22 points. Bob came first in Mathematics, but had only 9 points over-all. Charlie also finished with 9 points over-all. Who finished second in the English exam, and how many points were they awarded for it?

2. There are 2019 warriors sitting around a circular table. We may label them as Warrior 2 sitting to the left of Warrior 1, Warrior 3 sitting to the left of Warrior 2 etc. Learning that they are surrounded, they decide to commit suicide in a particular way. Warrior 1 kills Warrior 2, and passes the sword on to Warrior 3 who kills warrior 4 etc. Everytime a warrior kills the warrior closest to his left and the sword is passed on to the next living warrior. Which warrior is the last standing?

   A 557 **B** 863 **C** 1011 **D** 1991 **E** 2019

3. Ahmed took 7 Maths tests, each graded between 91 and 100 (inclusive, and only whole number grades). In no two tests did he get the same score. Ahmed noticed that after each test the average of the scores he had obtained was always a whole number. On the 7th test he got a 95. How much did he get on the 6th test?

   A 92 **B** 94 **C** 96 **D** 98 **E** 100

4. Amanda and Boris were caught by an evil guard. They were put in separate cells inside a forest. From her window, Amanda could see 12 trees, whilst Bob could see 8. They were both told that together they could see all the trees in the forest, and that no tree was common to both. The guard further informed them that every day he would ask a prisoner whether there are 18 or 20 trees in the forest. On Day 1 he will ask Amanda, and if she chooses to pass, on Day 2 Boris, and if he chooses to pass, on Day 3 Amanda etc. If anyone of them guesses wrongly they will both be executed. On the Day that one of them gets it correct, they are both set free. On which Day will they escape?

   A 2 **B** 3 **C** 9 **D** 11 **E** It is impossible to guarantee an escape

5. A colourless chessboard consists of m rows and n columns. We want to colour in the chessboard (with every square either black or white) in such a way that every row has at least 1 black square, all the rows have a distinct number of black squares, and a constant number of black squares in every column. For which kinds of integers m and n is this possible?

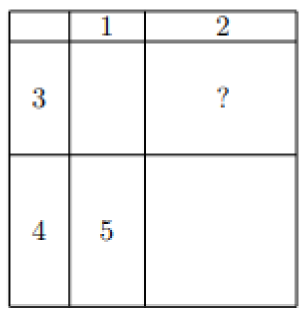
6. Three rows, containing three distinct positive whole numbers each, are written down on a paper. The numbers of each row sum to the same number n, whilst the product of the numbers in every row are different. Find the smallest n that satisfies these properties.

7. Two candles have exactly the same shape, but are made of different materials. The first one takes 4 hours to burn completely once lit and the other 5 hours. If we are to light both candles at the same time, at what time should we do this in order to ensure that at exactly 21:00 one candle is twice as tall as the other.

8. We have SEND + MORE = MONEY, where each letter represent a digit from 0 to 9 and $M \neq 0$. What is the numerical form of MONEY?

9. There are 20 classes in a certain school. Every class has a group of student welfare representatives, each consisting of 5 students. Rafael is a representative for his class, and he's the only representative of his school who has 4 female representatives in his group. Rafael notices that in 15 other groups there's a female majority, whilst for all 20 groups there are as many females as males. How many males are there in every group that has male majority?

10. Consider the following box of rectangles with the given areas. What's the missing area.



11. In a certain classroom with n students, one student has a bag containing 2019 1-pound coins, whilst the others have no money. Everytime two students meet, they split whatever money they have on them evenly if possible, and if not, they put away one 1-pound coin in a safe and split the rest evenly. After a while, the students notice that none of them have any money left and all has gone to the safe. What is the minimum value for n which makes this possible?

A 10 **B** 11 **C** 12 **D** 13 **E** 14

12. What is the smallest positive integer $n \geq 3$ which has the property that there are two distinct numbers in $\{1, 2, \ldots, n\}$ whose product equals the sum of the remaining $n - 2$ numbers?

13. Consider the four following statements:

A: It is not possible to partition the positive whole numbers into 2 disjoint sets X and Y such that the sum of any 2019 elements in X belongs to X, and the sum of any 2019 elements in Y belongs to Y, neither is it possible if we replace 2019 by 2020.

B: It is possible to partition the positive whole numbers into 2 disjoint sets X and Y such that the sum of any 2019 elements in X belongs to X, and the sum of any 2019 elements in Y belongs to Y, but it is not possible if we replace 2019 by 2020.

C: It is not possible to partition the positive whole numbers into 2 disjoint sets X and Y such that the sum of any 2019 elements in X belongs to X, and the sum of any 2019 elements in Y belongs to Y, but it is possible if we replace 2019 by 2020.

D: It is possible to partition the positive whole numbers into 2 disjoint sets X and Y such that the sum of any 2019 elements in X belongs to X, and the sum of any 2019 elements in Y belongs to Y, and it is still possible if we replace 2019 by 2020.

Which is true?

14. There are 2019 people in a room. The first person, Amber, has 1 rose, the second person has 2 roses, the third person 3 roses etc. Amber shakes hands once with every other person in the room, in a random order. If the person who Amber shakes hands with has more roses than her at the moment before the handshake, that person gives Amber one of their roses. What's the minimum number of roses Amber could have after she has shaken hands with everyone?

A 1 **B** 505 **C** 1009 **D** 1010 **E** 2017

15. Sam drives his car to work every morning. His path to work is divided into equal segments by 2019 traffic lights. The first light appears at the end of the first segment and the final light appears at the beginning of the final segment. One day Sam decides to make his path longer as follows: if he drives by a green traffic light he drives on, and if he's stopped by a red traffic light he returns to the previous traffic light (or his home, if the first light is red). Sam retreats from any given traffic light at most once. After doing this, he found that his total journey was twice as long as usual. How many traffic lights were red?

A 504 **B** 505 **C** 1009 **D** 1010 **E** 2018